PROCEEDINGS OF THE MATHEMATICS IN INDUSTRY STUDY GROUP

2023

Mathematics in Industry Study Group South Africa
MISGSA 2023

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The twentieth Mathematics in Industry Study Group (MISG) in South Africa was held at the African Institute for Mathematical Sciences, Muizenberg, Cape Town, from Monday 16 January to Friday 20 January 2023.

The total number of registered participants at the MISG was sixty-one. There were twenty-one Academic Staff, two Postdoctoral Fellows, thirty-two Graduate Students and six Industry Representatives. The invited guests were:

Neville Fowkes	University of Western Australia, Australia
Graeme Hocking	Murdoch University, Western Australia, Australia
Tim Myers	Centre de Recerca Matematica, Barcelona, Spain

There were two Postdoctoral Fellows

Oke Davies Adeyemo	North West University
Osman Noreldin	University of KwaZulu-Natal

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Sciences
North-West University
Rhodes University
Sol Plaatje University
University of Johannesburg
University of KwaZulu-Natal
University of South Africa (UNISA)
University of the Witwatersrand
University of Zululand

The International Universities which were represented, in addition to the Universities of the invited guests, were

Botswana International University of Science and Technology
University of Girona, Catalonia, Spain
University of Quebec in Montreal
Zayed Univesity, Dubai United Arab Emirates

There were virtual participants from the University of Oxford.

The MISG was officially opened on Monday morning by the Director of AIMS, Ulrich Paquet.

The MISG followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2022. Six problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by an academic moderator and one or more student moderators. The role of the academic moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results which were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Modelling Camp from Wednesday 10 January to Saturday 13 January 2023. The objective of the Graduate Modelling Camp is to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working collaboratively in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. Five problems were presented to the graduate students. The problems and the presenters were:

Covid mask design	Neville Fowkes University of Western Australia
Disturbance waves	David Mason University of the Witwatersrand
Decision support tool for optimal beer blending	Matthews Sejeso University of the Witwatersrand
Detecting oil and gas using sound waves	Erick Mubai University of the Witwatersrand
Green roofs to mitigate the Urban Heat Island	Gideon Fareo University of the Witwatersrand

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

The sponsors of the Graduate Workshop and the MISG were:

- Hermann Ohlthaver Trust
- African Institute for Mathematical Sciences
- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences
- School of Computer Science and Applied Mathematics, University of the Witwatersrand

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

STUDY GROUP

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PROBLEM STATEMENTS

Problem 1. Tourist attractions capping visitor numbers

Industry: Tourism Sector

Industry Representative: Dr Lombuso Precious Shabala
University of South Africa

Problem statement

The continuing tourism growth will eventually result in increased visitation to some destinations/ tourist attractions. Certain tourist attractions are limiting the number of visitors they welcome each day. The main reasons are to protect sensitive environments and provide a more enjoyable visitor experience by lessening the crowd. It must be noted that tourism caps have been around for decades and the pandemic travel patterns have encouraged new restrictions to take effect. It is perceived that visitation caps make for an inherently less flexible travel experience but as well as minimizing crowd and putting less strain on staff (which continues to be in short supply), limiting the number of visitors also helps to preserve and conserve natural resources.

It is vital for natural attractions to sustain the physical or ecological impact of visitors. The issue for managers surrounds the number of visitors that can be accommodated before the experience provided by the attraction is compromised. This challenge can be resolved determining the attraction's social carrying capacity (SCC) taking social comfort level (SCL) into account.

The Study Group was asked to investigate the following problem:

- Develop a viable mathematical model to determine the social carrying capacity of a tourist attraction. The capacity will assist management to determine the number of visitors that they can welcome each day. A decision should take into account the available infrastructure, activities, natural resources (biodiversity), visitors (day visitors versus overnight guests), accommodation available in the attraction which talks to the number of beds available versus occupied, and other key unique social comfort level variables.

- In addition, recalculate the visitor numbers that Mayeleti Nature Reserve should welcome per day. It is understood that only overnight visitor's numbers are cappied based on available accommodation on the site.

- Lastly, establish the limit visitor numbers per day for Mariepskop Nature Reserve, Bushbuckridge Nature Reserve and Unjaka Dam.

Problem 2. Sugar factory control strategy

Industry: Sugar cane processing

Industry Representative: Ricahard Loubser, Sugar Milling Research Institute,
University of KwaZulu-Natal, Durban

Problem statement

A sugar factory can be considered as a series of unit operations interspersed with buffer tanks and storage. The process needs to be controlled so that the buffers neither run empty nor overflow. The sugar cane that is processed is made up of fibre (insoluble solids), brix (soluble solids) and water. The exact amount of each of the components depends on the time of year, the variety of cane and the growing conditions. The throughput of each unit operation is limited by at least one of these components. In some units, a minimum throughout is required to maintain performance. At different times of the year, the bottlenecks in the factory will depend on the composition of the cane. Under optimum conditions, the unit operation that is the bottleneck at the time will run at or near capacity. The other operations will be controlled to keep the levels in the buffers from running dry or overflowing. Some processes (for example, clarification) are sensitive to rate of change of throughput rather than the throughput itself.

The questions are:

- Can the buffer tank theory be applied to the more complex multicomponent streams in sugar factories where the bottlenecking component changes along the process?

- How do we minimize the size of the buffer tanks (and hence the potential degradation) required to accommodate the normal fluctuations in operation?

- When is increasing the size of buffer tanks an economic alternative in terms of the value of through-put versus losses?

Problem 3. Enery management optimization for water distribution network tanks

Industry: Water treatment plants

Industry Representatives: David Albert, Alba Cabrera-Codony and Hector Monocles
Institute of the Environment, University of Girona, Spain

Problem Statement

Water treatment and distribution is one of the most energy consuming processes of all in the world. As an example, the percentage of energy usage in water treatment and distribution in Spain is 7% of all the energy consumed in Spain (Hardy et.al.,, 2010). This elevated energy consumption is the reason why optimizing energy in water distribution is a priority for water distribution networks managers.

The Barcelona water distribution network supplies water to more than 100 municipalities that have more than 4.5 million inhabitants. In order to supply water to all of this population more than 1000 km of pipes and 65 water storage tanks are used. This water distribution network has a tariff in which the energy costs vary with three different prices based on three different time slots.

The management of the water tanks of this network is done based on the assignment of three level set-points for each one of the three different time slots. These set-points define the level at which the water pumps that supply water to the tank have to stop. The tank water demand decreases the tank water level until it reaches the hysteresis level. This value defines the minimum water level at which the water pump starts working again.

The water distribution managers want to find a methodology to optimize the water set-point levels assigned for each one of the tanks to reduce energy costs.

Problem 4. Adsorption of multiple contaminants from a fluid stream

Industry: Carbon capture, climate change

Industry Representatives: Alba Cabrera-Codony
Institute of the Environment, University of Girona, Spain
Tim Myers
Centre de Recerca Matematica, Barcelona, Spain

Problem Statement

Perhaps the greatest danger currently facing mankind concerns environmental challenges and climate. In the most recent IPCC (Intergovernmental Panel on Climate Change) report [IPCC2021] on climate change it is stated that ``It is unequivocal that human influence has warmed the atmosphere, ocean and land. Observed increases in well-mixed greenhouse gas concentrations since around 1750 are unequivocally caused by human activities''. The link between climate change and human activity has been apparent for many years, it is therefore all the more tragic that first world countries could easily reduce emissions and achieve green energy targets. The well-known goal of maintaining the global temperature rise between $1.5 - 2^{\circ}\mathrm{C}$ can now only be achieved through drastic emission cuts combined with the active removal of greenhouse gases. Similarly the UN Sustainable Goal of a ``toxic free environment'' requires the removal of a multitude of ubiquitous pollutants.

One practical method of removing fluid based environmental contaminants is column sorption, either through absorption or adsorption. Column sorption involves passing a fluid through a tube filled with a material capable of capturing certain components of the fluid. The fluid enters at the inlet and contaminant components attach to the adsorbate. A standard laboratory experiment would involve a column of the order 20cm long and radius 5mm with a steady flow and contaminant escaping after around 15 minutes. Industrial columns are of the order of 5m tall and may run continuously for months with a constantly varying fluid intake.

Current adsorption models focus on the removal of a single containment, in which case the process is defined by an advection-diffusion equation linked to a sink model, typically an ODE, determined by the chemistry. The majority of studies involve numerical solutions. Mathematical analysis of single contaminant models is largely based on asymptotic reductions and travelling wave solutions. Processes where more than one pollutant is removed require multiple concentration equations and moving boundaries. Some pollutants may adsorb and then desorb due to competition with other components.

The goal of this study group problem is to develop and analyse models for the adsorption of a multiple contaminant fluid.

Problem 5. Modelling and optimization in the Robo Cup domain

Industry: Robotics

Industry Representative: Branden Ingram
University of the Witwatersrand, Johannesburg

Problem statement

The WITS RoboCup Team was founded in July 2019 and is affiliated with the RAIL Lab from the School of Computer Science and Applied Mathematics at the University of the Witwatersrand. The goal of the group is to develop a sustainable and competitive team that competes in the annual international RoboCup robotic competition. The objective is to develop teams which compete against each other in different leagues revolving around the game of Soccer. Robot Soccer has served as an excellent platform for testing learning scenarios in which multiple skills, decisions and controls have to be learned by a single agent, and agents themselves have to cooperate or compete.

Each robot has 22 degrees of freedom: six in each leg, four in each arm and two in the neck. The position of the ball is known throughout.

The first problem is to optimise a set of parameters which dictate the performance of a kicking policy to maximise the distance a ball is kecked.

The second problem is to optimize player positions in order to minimize defensive threats and maximise counterattack opportunities.

Problem 6. Temperature modelling in a furnace

Industry: Platinum processing

Industry Representative: John Atherfold
Opti-Num Solutions, Johannesburg

Problem Statement:

Pyrometallurgy deals with chemical reactions occurring at high temperatures. The purpose of the conversion Process in Pyrometallugy is to convert raw ore into matte and slag. Slag is molten rock and for the purposes of the process is a waste product that is discarded. Matte is a mixture of metal sulphides and is the final product of the conversion process, The matte is an intermediary product in the separation of metals from impurities, Conversion is used in the copper and lead industries as well as platinum group metals. The slag is less dense than the matte and hence floats on top of the bath.

In the furnace, process feed is fed into the top of the bath. In addition to this, a lance that feeds fuel coal, oxygen and other streams necessary for the reaction is submerged in the slag bath After a certain amount of time has passed, the matte and slag are tapped out of their respective tap holes. Ideally this is a continuous process, with constant feed and conversion and batch tappings.

All the exothermic chemical reactions occur in the slag bath. Matte droplets form in the slag bath as a result of the chemical reactions. Due to their higher density, these matte droplets drop out of the slag bath into the matte bath. The slag bath is hotter than the matte bath, and the slag bath is the source that adds heat to the matte bath through various mechanisms. Heat is lost through the sides of the furnace through indirect contact heat exchangers. The offgas from the chemical reactions also carries some heat with it as it moves out the top of the furnace on to the next stage of the process. The temperature of the furnace needs to be carefully controlled in order to maintain product quality as well as for safety considerations.

The Study Group is required to model the temperature in the furnace as a function of both temporal and special dimensions.